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Notes

TENSOR–PUFL: Boundary Condition Linked Codes*

Introduction

Finite-difference techniques have recently been applied to a wide variety of problems in gas [1–3], fluid [4, 5], and solid dynamics [6, 7].

Frequently, the variables associated with an experiment or natural phenomena simultaneously span such a large range of characteristic values that more than one physical description is required. Links between separate codes can increase the physically dissimilar regions which can be simultaneously simulated. A link also eliminates the development of one large code with numerous contingencies.

Brief Description of Codes

The TENSOR conservation equations [7] which use the complete stress tensor are cast in the two-dimensional, cylindrically symmetric, Lagrangian form. The TENSOR equations are given in Refs. [8] and [9], and a detailed description of the code is currently in preparation [10].

The PUFL equations [11] are cast in the "almost-Lagrangian" form for pipe geometry zones. Zones may have mass and energy sinks or sources, and auxiliary equations can relate these sources to consider the ablation and condensation processes. Details of the PUFL code are given in Refs. [11] and [12].

TENSOR–PUFL Link

TENSOR and PUFL are linked along a common boundary (for example, the inside radius of a pipe, Fig. 1). PUFL considers the material inside of the pipe, and TENSOR considers the walls of the pipe. The two codes provide boundary conditions to each other.

A TENSOR–PUFL calculation is initiated by first performing a PUFL cycle (or a set of PUFL cycles). Pressures, as a function of axial position at times when PUFL is entered and left, are then applied to the TENSOR boundary. Then a

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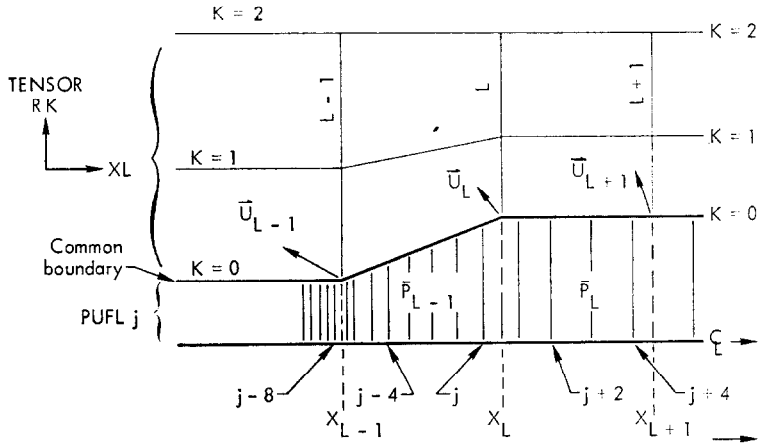

$\bar{P}_L$ = Pressure for TENSOR, surface weighted average, from PUFL zones between X_L amd X_{L+1}

$\vec{U}_L$ = Velocity of pipe wall at X_L for PUFL, from TENSOR.

FIG. 1. Geometry linking of TENSOR and PUFL.

TENSOR cycle (or a set of TENSOR cycles) is performed to obtain the PUFL boundary radii as a function of time and radial position.

Either a time-interval or a number-of-cycles criteria can be used to determine when to enter or leave each calculation. However, linear interpolations and extrapolations are presently used, and care must be taken to ensure that the

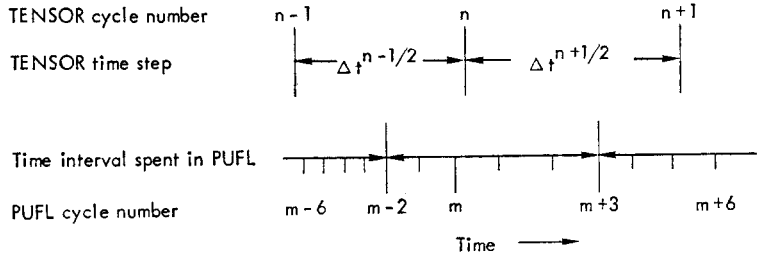

FIG. 2. Cycle linking of TENSOR and PUFL.

variation in radii or pressure is linear over the interval during which it is applied to the other code. Since PUFL zones and time steps are generally considerably smaller than those in TENSOR, PUFL is usually entered after each TENSOR cycle.

Figure 2 is a schematic diagram of one possible "leap-frog" technique for entering and leaving the two codes. It is shown in the figure that

(1) At PUFL time $t^{m-2} \geqslant$ TENSOR time $t^{n-1/2}$, TENSOR is entered at t^{n-1}.

(2) TENSOR calculates cycle $n-1$, thereby advancing to t^n; $\Delta t^{n+1/2}$ is also calculated.

(3) PUFL is entered at t^{m-2} and calculates cycles $m-2$ through $m+3$ to a time of $t^{m+3} \geqslant t^{n+1/2}$.

Boundary Condition Limitations

To insure the stability and the accuracy of the TENSOR–PUFL code, the stability and the accuracy of each code must be individually assured. Some PUFL calculations with rapidly changing radii were performed and compared with analytical solutions. These calculations indicated that when the change in volume per cycle is larger than 10%, the calculated solution diverges from the analytical solution. The PUFL time step restricts the anticipated change in volume per cycle to less than 10%.

Maenchen and Sack [8] compared TENSOR calculations with Blake's analytical solution [13] for a pressure profile of $P = P_0 e^{-\alpha t}$ applied to the inner boundary of a spherical elastic wedge. Following a procedure similar to Blake's, analytical solutions for the displacement, velocity, and pressure were obtained for a pressure profile of $P = P_0 e^{+\alpha t}$. TENSOR calculations similar to Maenchen and Sack's [8] which used various values of α, radial zone size (Δr), and time step (Δt), were considered.

The results indicated that an error, ϵ, whose upper limit is approximately $\epsilon = \alpha(\Delta r/4c)$ (where c is the local sound speed) occurs between the analytic and the numerical solutions. Note that this error is independent of the size of the time step (provided of course that the traditional zone size–sound speed criteria are satisfied). Transient errors somewhat larger than $\alpha(\Delta r/4c)$ are observed when the velocity passes through zero (in the $-\alpha$ calculation). However, after the velocity reestablishes itself in the negative direction, the errors can again be approximated by $\alpha(\Delta r/4c)$.

Calculations with both $+\alpha$ and $-\alpha$ show an initial overshoot and oscillation of the inner boundary as reported in Ref. [8]. One might think of this initial oscillation as being due to a pressure profile with an infinite $+\alpha$ applied for an infinitesimal length of time. The pressure profile applied to the TENSOR boundary by a shock passing through PUFL is of this nature, though $+\alpha$ is not infinite and the rise time is a few cycles. Thus, the initial overshoot conditions experienced in shock calculations are less violent than those shown in Ref. [8].

Scope of Applicability and Limitations

The applicability and limitations of the individual codes determine these factors for the link. Characteristic lengths and times need not be the same in the two calculations, but the desired accuracy of the boundary calculations should be considered.

The region to be simulated by TENSOR may be a cylindrically symmetric gas, liquid, or a plastic solid whose equation of state and material properties can be specified. The effects of heat transfer, changes of mass, and turbulence must be negligible, and the relative distortions of mass regions should not be severe.

The complete two-dimensional effects of a boundary layer flow cannot be considered by the pipe geometry of PUFL. However, some options for time- and space-dependent mass sources which are based on considerations of radial turbulent diffusion may be considered [12]. The effects of friction, heat transfer (by radiation, thermal convection, conduction, and arbitrarily specified sources), and mass sources (or sinks) may be considered.

It is assumed that the nonshock-related, thermal energy transfer to TENSOR is either negligible or that it reenters the PUFL region with the ablated mass. The total mass entering (or leaving) PUFL along a portion of the boundary must be negligible compared to that in the TENSOR zones along that portion of the boundary.

Boundary condition linking allows the predominent direction of motion in the two calculations to be perpendicular without zoning difficulties while maintaining the numerical advantages of Lagrangian codes. A slip surface has nearly these same advantages. However, one code with slip surfaces would require numerous contingencies to simulate simultaneously a solid and a high-energy gas.

Typical Solvable Problems

TENSOR–PUFL has been used to simulate (1) the effect of a nuclear detonation on a water-filled hole [14]; (2) a high-energy shock tube experiment [3]; (3) a nuclear-driven shock tube experiment [15]. The last two simulations were compared with experimental data.

Generally, TENSOR–PUFL can be used to simulate the flow of a material in a channel whose dimensions are dependent on the one-dimensional flow inside the channel. Some additional examples of typical problems are (1) the basic expanding or contracting balloon problem with mass entrainment or condensation; (2) the emplacement hole, stemming, and the surrounding region in an explosive experiment; (3) the flow of blood in an artery.

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